CHAPTER 6

SAMPLING PLANS

“No wise fish would go anywhere without a porpoise.”

The Mock Turtle

SAMPLING PLANS

1.0 INTRODUCTION

Sampling plans are hypothesis tests regarding product that has been submitted for an appraisal and subsequent acceptance or rejection. The product may be grouped into lots or may be single pieces from a continuous operation. A sample is selected and checked for various characteristics. For products grouped into lots, the entire lot is accepted or rejected. The decision is based on the specified criteria and the amount of defects or defective units found in the sample. Accepting or rejecting a lot is analogous to not rejecting or rejecting the null hypothesis in a hypothesis test. In the case of continuous production, a decision may be made to continue sampling or to check subsequent product 100%.

Sampling at the end of a manufacturing process provides a check on the adequacy of the quality control procedures of the manufacturing department. If the process has been controlled satisfactorily, the product would be accepted and passed on to the next organization or customer. If the process or quality controls have broken down, the sampling procedures will prevent defective products from going any farther. The manufacturing department, as part of the process or quality control program, may also use sampling techniques.

As processes become more refined and the process capabilities are known, the need for inspection becomes less important. The Inspection organization or end of the line appraisal function has three objectives that will be achieved in part through sampling techniques.

- Report the quality level of the manufacturing department to management. This is the primary objective of an inspection function.
- Provide adequate safeguards against the shipment of defective products.
- Assure that the manufacturing department has performed its quality functions properly.

2.0 METHODS FOR CHECKING PRODUCT

Selecting product for appraising quality characteristics can be done by a number of different methods. The six methods listed below are widely used.

- No checking
- 100% checking
- Constant percentage sampling
- Random spot checking
- Audit sampling (no acceptance and rejection criteria)
- Acceptance sampling based on probability

No checking may be warranted when the process capability is known and the probability of defective product is very small. A periodic audit to verify that conditions have not changed is a recommended practice when products are not checked on a routine basis. In some cases, incoming materials from various suppliers may not be inspected because the supplier has demonstrated outstanding quality capabilities.

When the process capability and the product quality level is not known, no checking usually results in increased costs for reworking defective product. When defective products are unknowingly shipped to the next using organization, subsequent operations may have to be halted to make corrections. When the risks involved are known, this technique will result in significant savings and increased product velocity. When the risks are not known, this technique may cause significant losses and problems to the company.

At the other extreme, product may be inspected 100%. In certain circumstances, 100% or even 200% checking may be necessary, particularly where lives are involved. In most routine processes, looking at each item is expensive, not always 100% effective and not necessary to assure product quality. One hundred percent checking is a sorting operation to separate good product from defective product. In addition, one hundred percent checking cannot be used when a destructive test is made. As the number of quality characteristics being checked increases, the effectiveness of the inspector decreases.

The unscientific sampling technique, known as the constant percentage sample, is a very popular procedure. This sounds like a logical procedure to many people. Why not make it easy and take a 10% sample from the lot? The problem with this method is that the sample taken from small lots may be too small and the sample taken from large lots may be too large. The inspection accuracy is not achieved for small lots and too much time and effort may be spent on large lots. Also, the sampling risks involved are not known. After a certain point, a larger sample will not yield more information. If the sample size is of sufficient size to determine the quality level and a decision can be reached as to accept or reject the lot, then further sampling would not be warranted.

Random spot-checking may sometimes be used when a process is in statistical control. The random check is used to verify that the process is in control and to report the product quality level. The sampling risks are not known, so this method will not guarantee that the outgoing quality will be at an acceptable level. This type of sampling may be used when a supplier has been certified as providing excellent quality products over some length of time or the process capability is so good that other methods of inspection are not necessary.

Audit sampling is sampling that is done on a routine basis, but acceptance criterion is not specified. A quality report is issued and the manufacturing organization will determine what action is to be taken if the material is not acceptable. Audit sampling is used where the manufacturing quality controls are known to be working correctly. The process capability must be known and the chance of defective products arriving at the inspection point must be very small.

Acceptance sampling based on probability is the most widely used sampling technique throughout industry. Many sampling plans are tabled and published and can be used with little training. The *Dodge-Romig Sampling Inspection Tables* are an example of published tables. Some applications require special unique sampling plans, so an understanding of how a sampling plan is developed is important. In acceptance sampling, the risks of making a wrong decision are known. When inspection is performed by attributes, (product is classified as good or defective) four types of acceptance sampling plans may be used, with lot by lot single sampling plans being the most popular. This is because they are easier to administer and implement than the other plans and they are very effective.

3.0 ACCEPTANCE SAMPLING PLANS

1) Lot by Lot - Single Sampling
2) Lot by Lot - Double Sampling
3) Continuous Sampling
4) Sequential Sampling

3.1 Lot by Lot - Single Sampling

- A lot size (N) of product is delivered to the quality check or inspection position.
- A sample size (n) is selected randomly from the lot.
- If the number of defects or defectives in the sample exceed the acceptance number (c or AN), the entire lot is rejected.
- If the number of defects or defectives in the sample do not exceed the acceptance number, the entire lot is accepted.
- Rejected lots are usually detailed 100% for the requirements that caused the rejection. In some cases the lot may be scrapped.
- Accepted lots and screened rejected lots are sent to their destination. The rejected lots may be submitted for re-inspection.

3.2 Lot by Lot - Double Sampling

- A lot size (N) of product is delivered to the quality check or inspection position.
- Two sample sizes (n_1, n_2) and two acceptance numbers (c_1, c_2 or AN_1, AN_2) are specified.
- A first sample of size n_1 is taken.
- If the number of defects or defectives in the first sample exceed c_2 , the lot is rejected and a second sample is not taken.
- If the number of defects or defectives in the first sample do not exceed c_1, the lot is accepted and a second sample is not taken.
- If the number of defects or defectives in the first sample are more than c_1 but less than or equal to c_2, a second sample n_2 is selected and inspected.
- If a second sample is inspected:

  a) and defects or defectives in combined first and second sample do not exceed c_2, the lot is accepted.

  b) and defects or defectives in combined samples exceed c_2, the lot is rejected.

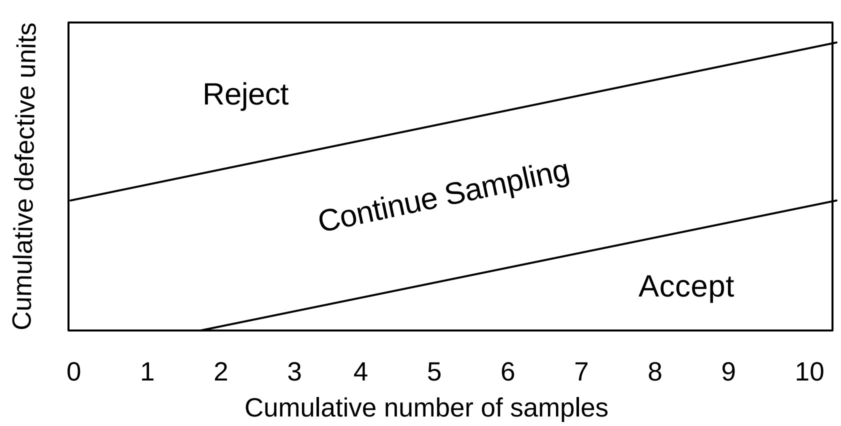
    i) Rejected lots are detailed or scrapped.
    ii) Accepted lots and detailed rejected lots are sent to their destination.

3.3 Continuous Sampling

- Continuous sampling is used where product flow is continuous and not feasible to be formed into lots.
- Two parameters are specified in a continuous sampling plan. The first is the frequency of checking f and the second is the clearing number i. The frequency f is expressed as 1/10, 1/20, 1/X, etc. and i is a number such as 20 or 50.
- When inspection begins, the product is checked 100% until i parts are found to be defect free. At this time, one out of X shall be inspected. If f = 1/10, then one out of 10 parts will be checked. The sampling will continue until a defect is found. When a defect is found, 100% inspection shall resume and the cycle starts over. When i parts are found to be defect free, the sample 1/X shall start again.
- In most cases, the inspector will not perform the 100% inspection. The inspector will mark the last sampled part and the manufacturing department will perform the 100% inspection or detailing operation.

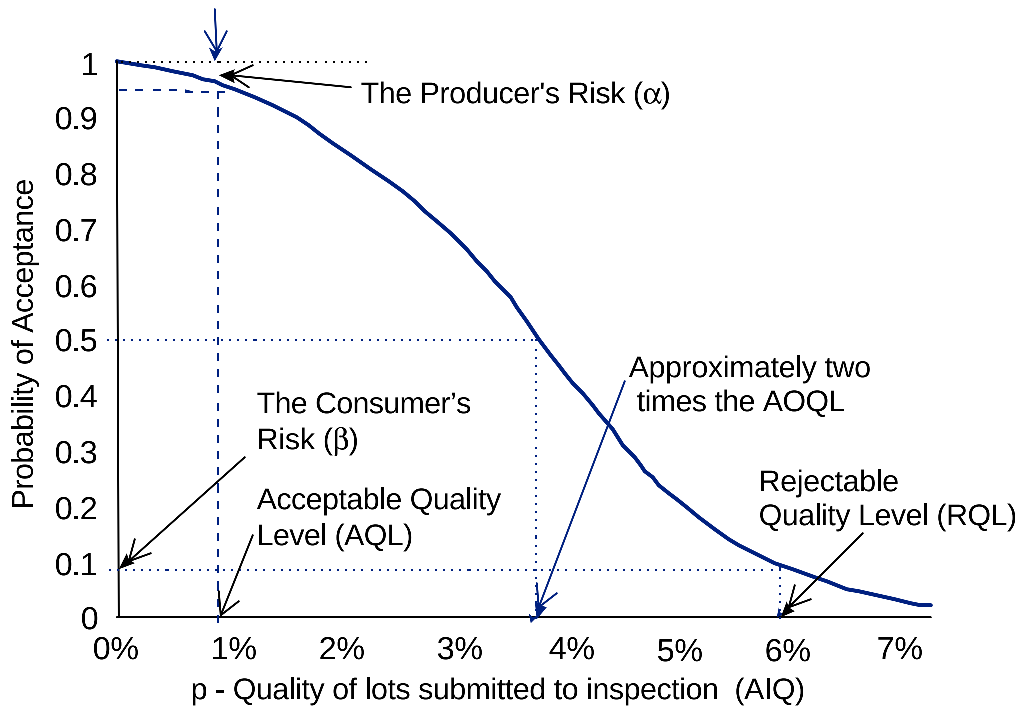
3.4 Sequential Sampling

- The inspector will select one part from the lot and check for the specified requirements.
- The part is classified as good or defective.
- A chart like the one shown below is specified for various sequential sampling plans. The required quality levels determine the acceptance, rejection, and continue sampling regions on the chart. The chart shows the inspector what decision to make after each sample is inspected. The lot will either be accepted rejected or another sample will be taken. This procedure is done on a lot by lot basis. The advantage of this type of sampling plan is that a decision could be made based on a relatively small sample.
- Rejected lots are detailed 100% (usually by the manufacturing department). Accepted and screened rejected lots are sent to their destination.



4.0 THE OPERATING CHARACTERISTIC CURVE (OC CURVE)

The Operating characteristic curve is a picture of a sampling plan. Each sampling plan has a unique OC curve. The sample size and acceptance number define the OC curve and determine its shape. The acceptance number is the maximum allowable defects or defective parts in a sample for the lot to be accepted. The OC curve shows the probability of acceptance for various values of incoming quality.



An OC curve is developed by determining the probability of acceptance for several values of incoming quality. Incoming quality is denoted by p. The probability of acceptance is the probability that the number of defects or defective units in the sample is equal to or less than the acceptance number of the sampling plan. The AQL is the acceptable quality level and the RQL is rejectable quality level. If the units on the abscissa are in terms of percent defective, the RQL is called the LTPD or lot tolerance percent defective. The producer's risk (α) is the probability of rejecting a lot of AQL quality. The consumer's risk (β) is the probability of accepting a lot of RQL quality.

There are three probability distributions that may be used to find the probability of acceptance. These distributions were covered in the Basic Probability chapter and are reviewed here.

- The hypergeometric distribution
- The binomial distribution
- The Poisson distribution

Although the hypergeometric may be used when the lot sizes are small, the binomial and Poisson are by far the most popular distributions to use when constructing sampling plans.

4.1 Hypergeometric Distribution

The hypergeometric distribution is used to calculate the probability of acceptance of a sampling plan when the lot is relatively small. It can be defined as the true basic probability distribution of attribute data but the calculations could become quite cumbersome for large lot sizes.

The probability of exactly x defective parts in a sample n:

$$P(x) = \frac{\binom{n}{x}\binom{N-n}{n-x}}{\binom{N}{n}}$$

4.2 Binomial Distribution

The binomial distribution is used when the lot is very large. For large lots, the non-replacement of the sampled product does not affect the probabilities. The hypergeometric takes into consideration that each sample taken affects the probability associated with the next sample. This is called sampling without replacement. The binomial assumes that the probabilities associated with all samples are equal. This is sometimes referred to as sampling with replacement although the parts are not physically replaced. The binomial is used extensively in the construction of sampling plans. The sampling plans in the *Dodge-Romig Sampling Tables* were derived from the binomial distribution.

The probability of exactly x defective parts in a sample n:

$$P(x) = \binom{n}{x} p^{x}(1-p)^{n-x}$$

The symbol p represents the value of incoming quality expressed as a decimal. (1% = .01, 2% = .02, etc.)

4.3 Poisson Distribution

The Poisson distribution is used for sampling plans involving the number of defects or defects per unit rather than the number of defective parts. It is also used to approximate the binomial probabilities involving the number of defective parts when the sample (n) is large and p is very small. When n is large and p is small, the Poisson distribution formula may be used to approximate the binomial. Using the Poisson to calculate probabilities associated with various sampling plans is relatively simple because the Poisson tables can be used. The Thorndike chart, which will be discussed later, is a valuable aid in the construction of sampling plans using the Poisson distribution.

The probability of exactly x defects or defective parts in a sample n:

$$P(x) = \frac{e^{-np}(np)^{x}}{x!}$$

The letter e represents the value of the base of the natural logarithm system. It is a constant value (e = 2.71828).

4.4 An OC Curve Using the Binomial Distribution

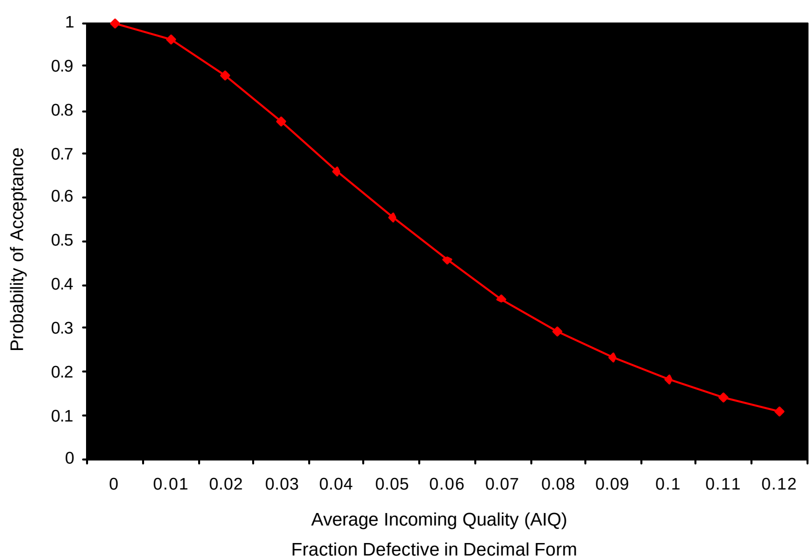
For any value of the AIQ, the probability of acceptance is the probability of c or fewer defective parts where c is the acceptance number for the sampling plan. The probability of acceptance is usually expressed as a decimal rather than as a percentage. It is represented by the symbol P_a. The letter n represents the sample size. For a sampling plan with an acceptance number of 0, $P_a = P(0)$. For a sampling plan with an acceptance number of 1, $P_a = P(0) + P(1)$.

$$P_a = P(0) + \ldots + P(c)$$

The probability of acceptance is calculated for a sampling plan with n = 30 and c = 1.

p = AIQ	**P(0)**	**P(1)**	**P_a**
.01	$\binom{30}{0}(.01)^0(.99)^{30} =$ .740	$\binom{30}{1}(.01)^1(.99)^{29} =$ .224	.964
.02	$\binom{30}{0}(.02)^0(.98)^{30} =$ .545	$\binom{30}{1}(.02)^1(.98)^{29} =$ .334	.879
.03	$\binom{30}{0}(.03)^0(.97)^{30} =$ .401	$\binom{30}{1}(.03)^1(.97)^{29} =$ .372	.773
.04	$\binom{30}{0}(.04)^0(.96)^{30} =$ .294	$\binom{30}{1}(.04)^1(.96)^{29} =$ .367	.661
.05	$\binom{30}{0}(.05)^0(.95)^{30} =$ .215	$\binom{30}{1}(.05)^1(.95)^{29} =$ .339	.554
.06	$\binom{30}{0}(.06)^0(.94)^{30} =$ .156	$\binom{30}{1}(.06)^1(.94)^{29} =$ .299	.455
.07	$\binom{30}{0}(.07)^0(.93)^{30} =$ .113	$\binom{30}{1}(.07)^1(.93)^{29} =$ .256	.369
.08	$\binom{30}{0}(.08)^0(.92)^{30} =$ .082	$\binom{30}{1}(.08)^1(.92)^{29} =$ .214	.296
.09	$\binom{30}{0}(.09)^0(.91)^{30} =$ .059	$\binom{30}{1}(.09)^1(.91)^{29} =$ .175	.234
.10	$\binom{30}{0}(.10)^0(.90)^{30} =$ .042	$\binom{30}{1}(.10)^1(.90)^{29} =$ .141	.183
.11	$\binom{30}{0}(.11)^0(.89)^{30} =$ .030	$\binom{30}{1}(.11)^1(.89)^{29} =$ .112	.142
.12	$\binom{30}{0}(.12)^0(.88)^{30} =$ .022	$\binom{30}{1}(.12)^1(.88)^{29} =$ .088	.110

The Operating Characteristic Curve for n = 30 and c = 1 using the Binomial Distribution



4.5 An OC Curve Using the Poisson Distribution

The Poisson distribution is used to compute the probability of acceptance for defects per unit sampling plans. It may also be used to approximate binomial probabilities and compute the probability of acceptance for fraction defective sampling plans.

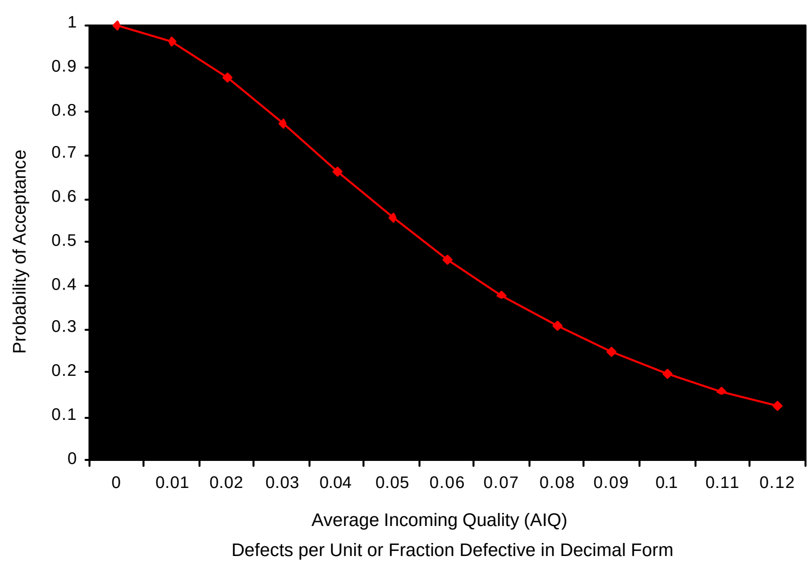
For the sampling plan n = 30 and c = 1, c may be in terms of number of defects or in terms of number of defective parts. If c is in terms number of defects, the AIQ or abscissa on the OC curve is in terms of defects per unit. The acceptance number for the sampling plan n = 30 and c = 1 may either be 1 defect or 1 defective part.

The Poisson formula, $P(x) = \dfrac{e^{-np}(np)^x}{x!}$, is used to compute the probabilities of acceptance.

p = AIQ	**m= np**	**P(0)**	**P(1)**	P_a
.01	.30	.741	.222	.963
.02	.60	.549	.329	.878
.03	.90	.407	.366	.772
.04	1.2	.301	.361	.663
.05	1.5	.223	.335	.558
.06	1.8	.165	.298	.463
.07	2.1	.122	.257	.380
.08	2.4	.091	.218	.308
.09	2.7	.067	.181	.249
.10	3.0	.050	.149	.199
.11	3.3	.037	.122	.159
.12	3.6	.027	.098	.126

For all practical purposes, the probabilities of acceptance are the same as those obtained with the binomial formula. There are some minor differences. For this example, the differences increase slightly as the curve approaches the tail.

The Operating Characteristic Curve for n = 30 and c = 1 using the Poisson Distribution



5.0 THE AVERAGE OUTGOING QUALITY CURVE

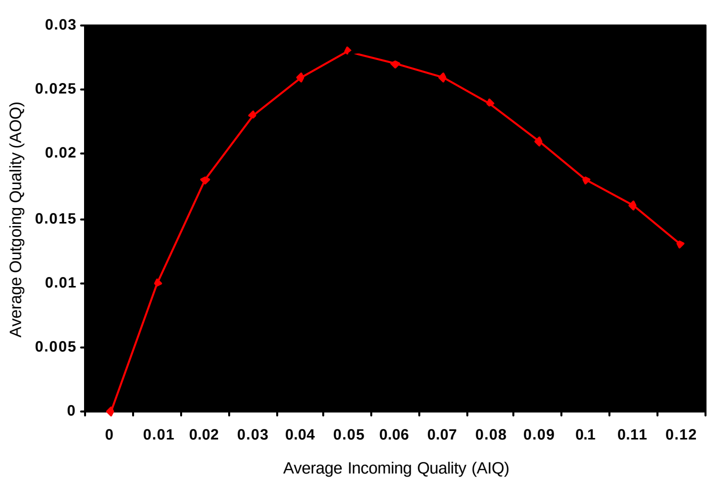
For every acceptance sampling plan, the outgoing quality will be somewhat better than the incoming quality because a certain percent of the lots will be rejected and detailed. The Average Outgoing Quality (AOQ) curve shows the relationship between incoming and outgoing quality. The AOQ and OC curve, when used together, describe the characteristics of the sampling plan and the risks involved.

$$AOQ = (p)(P_a)\frac{(N-n)}{N}$$

The letter p is the incoming quality level (AIQ) and P_a is the probability of acceptance. The abscissa for the AOQ curve is the same as for the OC curve. The Average Outgoing Quality Limit (AOQL) is, on average, the maximum value of the AOQ. When the lot (N) is very large, the expression (N - n)/N approaches 1 and may be omitted. For this example, the lot size is 5000 pieces. Therefore, (N - n)/N = (5000 - 30)/5000 = .994 ≈ 1 and is dropped out. For this AQL curve, the Probabilities of acceptance (P_a) for the sampling plan n = 30 and c = 1 are based on the binomial distribution.

p = AIQ	**P_a**	**AOQ = (p)(P_a)**
.01	.964	.010
.02	.879	.018
.03	.773	.023
.04	.661	.026
.05	.554	.028
.06	.455	.027
.07	.369	.026
.08	.296	.024
.09	.234	.021
.10	.184	.018
.11	.143	.016
.12	.110	.013

AOQ Curve



The AOQL is approximately .028 at an incoming quality level of .05. The AOQL is the outgoing quality level at the crest of the curve.

6.0 PROBABILITY NOMOGRAPHS

A Nomograph is a paper slide rule that helps to simplify certain computations. Many of the same computations may be made more elegantly on a computer. Nomographs were popular before there were computers. Since computers are not allowed at the CQE exam, the nomographs may come in handy.

6.1 Binomial Nomograph

Ken Larson of the AT&T Company developed the binomial nomograph. The nomograph greatly simplifies and reduces the computational burden involved with solving binomial problems. It permits direct solution of some problems not otherwise directly solvable except by approximation or computer. The nomograph contains the cumulative binomial probabilities $P(0) + \ldots + P(c)$, and may be used to determine sample sizes and acceptance numbers for acceptance sampling plans. The probability of accepting a lot is the probability of c or fewer defective parts. The nomograph is used for fraction defective sampling plans.

The values for the operating characteristic (OC) curve are obtained directly from the nomograph.

6.2 Thorndike Chart

The Thorndike chart was developed by Frances Thorndike of Bell Laboratories in 1926. It is a nomograph of the cumulative Poisson probability distribution. Like the binomial nomograph, it may also be used to determine sample sizes and acceptance numbers for sampling plan applications. The Thorndike chart is used for the following sampling plans:

- Defects per unit sampling plans.
- Approximation to the binomial probabilities for fraction defective sampling plans.

The abscissa on the Thorndike chart is np. The ordinate is the probability of c or fewer occurrences. The curved lines in the body of the chart represent the cumulative number of occurrences or successes that are of interest. The curved lines also represent the acceptance numbers in a sampling plan. The Thorndike chart may be used as an alternative to the Poisson tables when determining cumulative probabilities.

7.0 SAMPLING PLAN CONSTRUCTION

Sampling plans may be developed to meet certain criteria and to insure that the specified outgoing quality levels are met. In the construction of a lot by lot single sampling plan, four parameters must be determined prior to determining the sample size and acceptance number.

The parameters are:

- The Acceptable Quality Level (AQL)
- Alpha (α), The Producers Risk
- The Rejectable Quality Level (RQL)

- Beta (β), The Consumers Risk

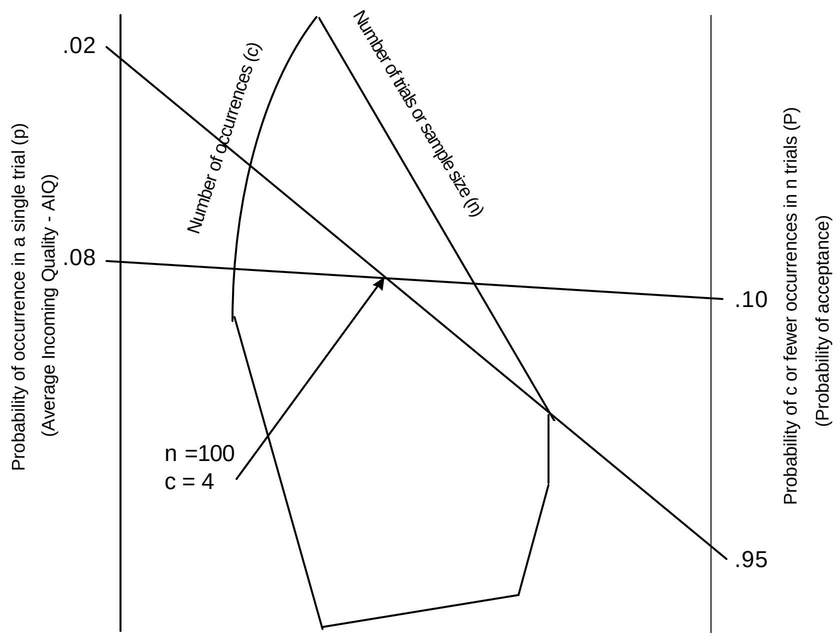
The objective is to find a sample size and acceptance number whose OC curve meets the above parameters. The sampling plan will have a 1 - α chance of being accepted when the incoming quality is at the AQL level and a β chance of being accepted when the incoming quality is at the RQL level. An easy way to find the sample size and acceptance number is to use a the binomial nomograph or the cumulative Poisson nomograph called the Thorndike chart. Copies of the binomial nomograph and Thorndike chart are included in the appendix. They will be provided separately if you have the computer based version of QReview. They are also included in various textbooks.

Sampling plans will be constructed using both the binomial nomograph and the Thorndike chart. The AQL, RQL, α, and β must be specified. They may be determined by your customer, special studies, or past experience. The most common values to use for α and β are .05 and .10 respectively. The following values are used for the binomial nomograph and Thorndike chart examples:

AQL = 2% (.02), α =.05, RQL = 8% (.08), β = .10

7.1 Sampling Plan Construction Using the Binomial Nomograph

The AIQ is on the left scale and the probability of acceptance is on the right scale. The semi vertical lines on the nomograph represent the sample sizes and the semi horizontal lines are the acceptance numbers. Draw a line from the AQL (.02) to its probability of acceptance (.95) and a line from the RQL (.08) to its probability of acceptance (.10). The intersection will yield the sample size and acceptance number. Do not interpolate: Find the closest sample size and acceptance number to the intersection point. Both the sample size and acceptance numbers must be integers.



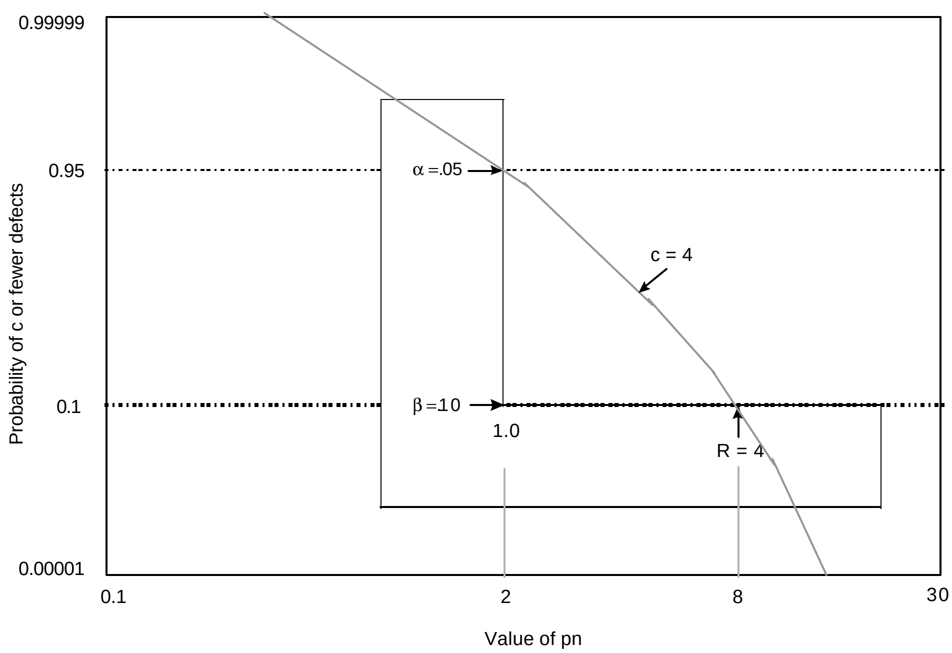
The sample size is 100 and the acceptance number is 4.

7.2 Sampling Plan Construction Using the Thorndike Chart

The Thorndike chart can also be used to find the sample size and acceptance number for the plan. To assist in the task, a tool called an L will be used. The L may be modified for any value of β. The inside horizontal scale on the L must be lined up with β and α is to coincide with 1 - α on the vertical axis. The left or vertical side of the L coincides with the probability of acceptance on the Thorndike chart.

R is the ratio of the RQL to the AQL (R = RQL/AQL). R is computed and marked with an arrow as shown on the diagram.

The L is moved across the chart until one of the acceptance number curves goes through both the α value on the inside vertical scale and the R value on the inside horizontal scale. The best fitting acceptance number curve is the one to select. The curve represents the acceptance number (c) for the plan. The curve c = 4 goes through both the points α = .05 and R = 4 (R = .08/.02 = 4).



The sample size is determined as follows: At the corner of the L where the value is 1, drop a straight line to the pn scale on the abscissa of the Thorndike chart. The value of pn is 2. The value of p at this point is the AQL (AQL = .02). If pn =2 and p = .02 then n = 2/.02 = 100. Also a straight line can be dropped from R to the pn scale. The value of pn is 8. The value of p at this point is the RQL (RQL = .08). If pn = 8 and p = .08 then n = 8/.08 = 100. The same answer is obtained at either point.

The sample size is 100 and the acceptance number is 4.

Both the binomial nomograph and the Thorndike chart give the same sample size and acceptance number. In some cases, there may be minor variations between the two methods.

8.0 GLOSSARY OF TERMS

- ***Acceptance Number:*** The maximum allowable defects or defectives in a sample for the lot to be accepted. (Acceptance number = AN or c)
- ***AIQ - Average Incoming Quality:*** This is the average quality level going into the inspection point. The inspection data and subsequent report reflects this number. The AIQ is the abscissa on the OC and AOQ curves.
- ***AOQ - Average Outgoing Quality:*** The average quality level leaving the inspection point after rejection and acceptance of a number of lots. If rejected lots are not checked 100% and defective units removed or replaced with good units, the AOQ will be the same as the AIQ.
- ***AOQ Curve - Average Outgoing Quality Curve:*** The curve or graph that shows the Average Outgoing Quality for various values of incoming quality.
- ***AOQL - Average Outgoing Quality Limit:*** The maximum value of the AOQ. If the sampling procedures are followed, the outgoing quality will, on average, not be worse than the AOQL.
- ***AQL - Acceptable Quality Level:*** The quality level for which there is a high probability of accepting the lot. The AQL is also defined as the maximum fraction defective or defects per unit that can be considered satisfactory as a process average.
- ***Attribute data:*** Although measurements may be taken, they are not recorded on the data sheet. The product is classified as good or defective. (Discrete data)
- ***Consumers risk (b):*** The probability of accepting a lot with a high number of defective units. β is usually set at .05 to .15 with .10 as the common value. β can also be defined as the probability of accepting a lot of RQL or LTPD quality.
- ***Lot**:* A collection of individual pieces from a common source, possessing a common set of quality characteristics and submitted as a group for acceptance at one time. (Lot size = N)
- ***LTPD - Lot Tolerance Percent Defective:*** This is the value of incoming quality where it is desirable to reject most lots. The quality level is unacceptable. This is the RQL expressed as a percent defective.
- ***OC Curve - Operating Characteristic Curve:*** The curve or graph shows the probability of a lot being accepted for various values of incoming quality.
- ***Power of Test (1 - b):*** This is the probability of rejecting a lot when the incoming quality is at the RQL level.
- ***Process average:*** The normal or stable quality level of a product or process for a specified period of time. The quality level may be expressed as a fraction defective, percent defective, or defects per unit.

- ***Producers risk (α):*** The probability of rejecting a *good* lot. This is usually set at .01 to .10 for most sampling plans. The symbol α can also be defined as the probability of rejecting a lot of AQL quality.

- ***Random sample:*** A sample selected in such a manner that any piece of product in the lot has an equal chance of being chosen.

- ***RQL - Rejectable Quality Level:*** The generic term for the incoming quality level for which there is a low probability of accepting the lot. The quality level is substandard.

- ***Sample:*** A subset of a lot selected to be inspected. (Sample size = n)

- ***Sampling plan:*** The procedure that specifies the number of units to be selected from a lot or batch for appraisal. The sample size and acceptance number describes each unique sampling plan.

- ***Variables data:*** Actual measurements are taken and recorded. (Continuous data)