# CHAPTER 7

## CONTROL CHARTS



**“I don't want to achieve immortality through my work. I want to achieve it by not dying.”**

**Woody Allen**

# CONTROL CHARTS

## 1.0 INTRODUCTION

Two categories of charts are typically used in manufacturing and service industries. They are performance charts and control charts. The charts may contain identical data, have upper and lower limits and look the same. The factor that differentiates the two charts is not how they look, but how they are used. A performance chart may display quality results, efficiency or both. It is used to display current performance and trends over time. Performance charts are also used to show group performance and to promote competition between various groups. A performance chart is not a control chart although it may contain upper and lower limits. It is not intended to exhibit statistical control although in some cases it may do so. It is intended simply as a display of performance.

A control chart is used as a tool to continuously monitor and, if needed, make adjustments to the product or process. It does not measure performance. On a performance chart, eight consecutive points on the plus side of the nominal may indicate sustained good performance. The same pattern on a control chart indicates that the process is not following a normal pattern of variation and is out of control. The purpose of a control chart is to indicate whether or not a process is in statistical control. Statistical control means that the plotted points follow a pattern of variation consistent with the areas under the normal curve.

For a chart to be called a control chart it should work much the same way as a thermostat in a home. The thermostat is set at a nominal value and continuously monitors temperature. If the home becomes cold, the thermostat will immediately make an adjustment by turning on the furnace. When the home reaches a certain temperature the thermostat will shut off. Its job is to maintain normal variation around the nominal value. This is an example of automated process control.

Control charts work best when they are automated. A few years ago, before computers, the implementation and maintenance of control charts was a manual task. The charts needed to be interpreted by humans and were subject to human error. In some operations, manual control charts may be appropriate and efficient, but if the system can be automated, that is the way to go.

Although more automation will be used in the future, it is important that quality practitioners understand how to interpret and manually construct the various types of control charts.

## 2.0 CONTROL CHARTS CONCEPTS

Control charts are a means of graphing variation patterns from process or product characteristics so that corrective action may be taken if required. When a process is in statistical control, a control chart will display known patterns of variation. When the control chart points deviate from these known patterns, the process is considered to be out of control. A control chart is to be used like a gauge to indicate when adjustments are required.

As in the thermostat example above, when the data values follow certain patterns, no adjustments in the process are necessary. When the data values stray from the normal pattern, adjustments or other actions are required to bring the process back into control. Control charts are the major statistical tools used in statistical process control (SPC) and statistical quality control (SQC). SPC and SQC are the application of various statistical methods to make a process or product behave the way it is intended to behave.

Control charts were one of the first statistical techniques introduced in statistical quality control. Dr. Walter A. Shewhart of AT&T Bell Laboratories developed the charts in 1924. The original charts for variables data, x bar ($\bar{x}$) and R charts, were called Shewhart charts. The control chart concept was introduced in his book *The Economic Control of Manufactured Product* published in 1931.

The control chart distinguishes between normal and non-normal variation through the use of statistical tests and control limits. The control limits are calculated using the rules of probability so that when a point is determined to be out of control, it is due to an assignable cause and not due to normal variation. Points outside the control limits are not the only criteria to determine out of control conditions. All points may be inside the limits and the process may still be out of control if it does not display a normal pattern of variation. Zone tests are used to determine out of control conditions. Zone tests are hypothesis tests in a modified form. They are used to test if the plotted points are following a normal pattern of variation.

Control charts are statistical tools that can help in establishing process capabilities, identifying problems that cause out of control conditions and maintain control of product and process quality. The control chart will not do these things by itself. For a control chart to be effective, some action must be taken as a result of the chart pattern. When the process average is centered where it is supposed to be, and the variability displays a normal pattern, the process is considered to be in control. A normal pattern means that the process is aligned with the probabilities of the normal distribution. Large abnormal variability and unnatural patterns indicate out of control conditions. Out of control conditions usually have assignable causes that must be investigated and resolved.

There are two types of control charts, the variables control chart and the attributes control chart. The variables charts use actual measurements as data and the attribute charts use percentages or counts.

## 3.0 VARIABLES CONTROL CHARTS

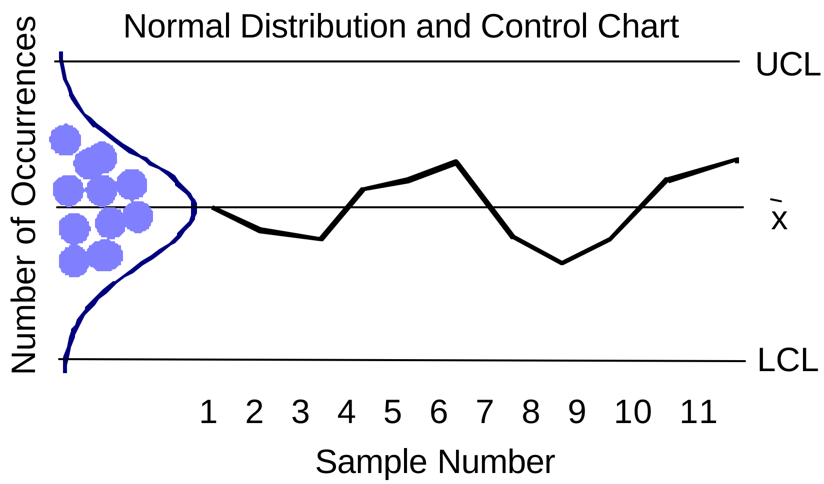
### 3.1 The x Bar ($\bar{x}$) and R Charts

Charts using data obtained from measurements are the most powerful of all control charts. Two charts that are used together to chart variables data are called $\bar{x}$ and R charts. The sample average is represented by $\bar{x}$ and R is the range. The range is the difference between the highest and lowest number in the sample. The charts are very effective indicators of problems in the process and also indicate when the problems have been cleared.

### 3.2 Normal Distributions and Control Charts

A variables control chart is a picture of a sampling distribution of averages. It is not a plot of individual data values. The chart is just another way of displaying the bell shaped curve of the normal distribution for a distribution of averages. If a normal distribution of averages is turned on its side and the mid-point and ± 3 sigma points are extended horizontally, a control chart is created.

Points on an $\bar{x}$ chart always follow a normal pattern of variation. The points are averages of small samples. The Central Limit Theorem states that a distribution of averages tends to follow a normal distribution pattern.




3.3 Data Collection and Calculations for Constructing $\bar{x}$ and R Charts

- Samples of 4, 5, or 6 readings are to be taken and recorded on an appropriate form. Five is the most common and economical sample size. Five provides a good economical balance between confidence and cost.

- The average ($\bar{x}$) of each sample is calculated by adding the measurements in the sample and dividing by the number of measurements. These averages tend to follow a normal distribution pattern even if they come from a non-normal process.

- The range (R) is computed by subtracting the smallest measurement from the largest measurement in each sample.

- The centerline for the $\bar{x}$ chart is calculated by averaging all the sample averages. This is designated as the average of averages or $\bar{\bar{x}}$.

- The centerline for the R chart is calculated by averaging the range values and designating this value as $\bar{R}$.

- Control limits for the $\bar{x}$ chart are calculated as follows:

$$\text{Upper Control Limit } (UCL_{\bar{x}}) = \bar{\bar{x}} + A_2\bar{R}$$
$$\text{Lower Control Limit } (LCL_{\bar{x}}) = \bar{\bar{x}} - A_2\bar{R}$$

- Control limits for the R chart are calculated as follows:

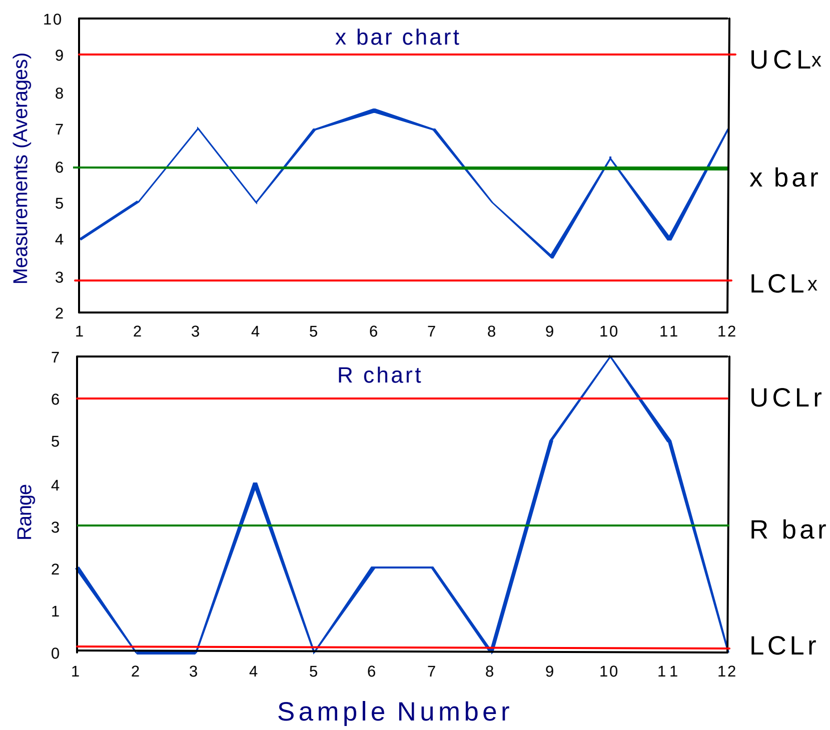
$$\text{Upper Control Limit } (UCL_r) = D_4\bar{R}$$
$$\text{Lower Control Limit } (LCL_r) = D_3\bar{R}$$

The factors $A_2$, $D_3$, and $D_4$ are listed in the table of factors.

3.4 Construction of the $\bar{x}$ and R Chart

Both charts are drawn on one sheet of paper. For the charts to be effective, it is important that they are viewed at the same time. A vertical scale should be selected to permit a full range of chart values and also prevent overlapping or extending off the graph. A piece of plain graph paper can be used to plot the charts. Charts also may be customized on special forms for specific applications.

The $\bar{x}$ chart is drawn on the upper half of the graph paper and the R chart is drawn on the lower half. The R chart is started at or near the bottom of the sheet using an appropriate scale to cover nearly all possible range values. The upper control limit for the R chart should be slightly below the midpoint of the graph paper. The centerline for the $\bar{x}$ chart is drawn at approximately the middle of the upper half of the sheet and scaled accordingly to make sure that the upper and lower control will fit in the upper half of the sheet. The control limits are to be labeled $UCL_x$, $LCL_x$, $UCL_R$, and $LCL_R$.



## 4.0 ATTRIBUTE CONTROL CHARTS

Charts that use counts or percentages are somewhat less powerful tools than variables charts but can be used to great advantage if they are understood and used properly. Attribute charts are used where it is impossible, difficult or uneconomical to make a numerical measurement. In some cases attribute charts may be preferred as the chart of choice. Attribute charts have the distinct advantage of combining multiple characteristics on one chart.

### 4.1 The p Chart

The most frequently used attribute control chart is the p or percent defective chart. Even though many quality characteristics may be combined on a p chart, it will be easier to interpret if the characteristics are limited to the few that are the most troublesome.

### 4.2 Data Collection and Calculations for p Chart

- Sample sizes are larger for attributes control charts than for variables control charts. Attribute sample sizes may range from 50 to 100 or more. The average sample size $\overline{n}$ is used in the formulas for the control limits.

- The percent defective (p) for each sample is calculated by dividing the total number of defective units in the sample by the total sample size. This is the percent of *defective* units rather than the percent of *defects*. A defective unit may contain one or more defects.
- The centerline for a p chart is calculated by dividing the total number of defective units in all samples by the total units checked in all samples. The average percent defective is called p-bar ($\overline{p}$).

- The control limits are calculated by the following formulas:

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{\overline{n}}}$$

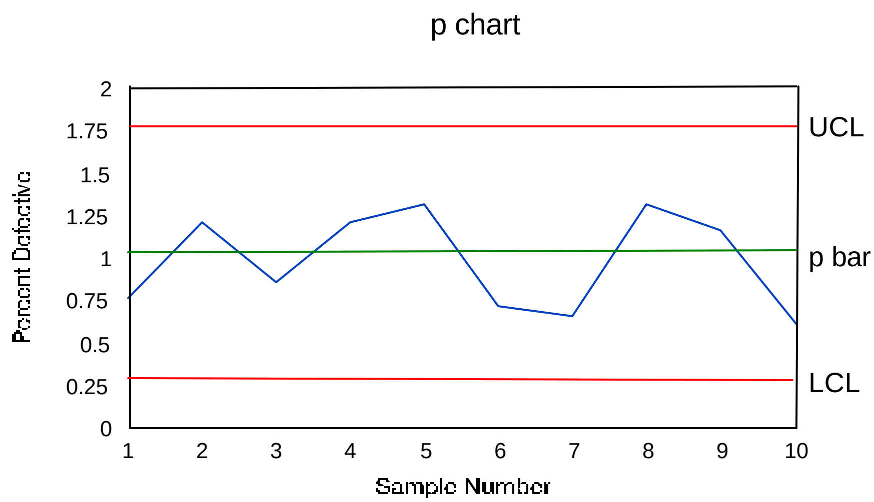
$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}\,(1-\overline{p})}{\overline{n}}}$$

- The lower control limit (LCL) is set to zero if the calculated value is negative.

### 4.3 Construction of the p Chart

Since only one chart is placed on the graph paper, a vertical scale is constructed by placing the centerline $\overline{p}$ at the center of the sheet, the upper control limit (UCL) near the top of the sheet and the lower control limit (LCL) near the bottom of the sheet.

The centerline is indicated by a solid line and is labeled $\overline{p}$. The upper and lower control limits are drawn as dashed or dotted lines and labeled UCL and LCL respectively. The samples do not need to be the same size for a p chart. The average sample size $\overline{n}$ is used in the control limit formulas. The largest sample used should not be more than twice the average sample size.




## 4.4 The np Chart

The np chart is used to plot the number of defectives when the sample sizes are all equal. If the sample sizes are not all equal, then a p chart is to be used instead of a np chart.

$$\text{Centerline} = n\overline{p}$$

$$\text{Upper Control Limit} = \text{UCL} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})}$$

$$\text{Lower Control Limit} = \text{LCL} = n\overline{p} - 3\sqrt{n\overline{p}(1-\overline{p})}$$

## 4.5 The u Chart

The u chart is used to plot defects per unit. The sample sizes need not be equal. This type of chart is used for complicated assemblies where the possibilities for defects are infinite but a constant sample size is not possible.

$$\text{Centerline} = \overline{u} = \text{Average defects per unit}$$

$$\text{Upper Control Limit} = \text{UCL} = \overline{u} + 3\sqrt{\frac{\overline{u}}{\overline{n}}}$$

$$\text{Lower Control Limit} = \text{LCL} = \overline{u} - 3\sqrt{\frac{\overline{u}}{\overline{n}}}$$

4.6 The c Chart

The c chart is used to plot the number of defects when the sample sizes are equal. This chart is used where the possibilities for defects are infinite and the probability of getting a defect is very small.

$$\text{Centerline} = \bar{c} = \text{Average number of defects in the sample}$$
$$\text{Upper Control Limit} = UCL = \bar{c} + 3\sqrt{\bar{c}}$$
$$\text{Lower Control Limit} = LCL = \bar{c} - 3\sqrt{\bar{c}}$$

## 5.0 TESTS FOR UNNATURAL PATTERNS

A control chart is considered to have unnatural patterns if the plotted points follow any of the combinations shown on the zone test table below. The tests are used for both variables and attribute type control charts. In applying the tests, consider each half of the chart separately.

5.1 Zone Tests

Each zone width is one standard error for a variables chart and one standard deviation for an attributes chart.

| | | |
|---|---|---|
| | | Single point above upper control limit |
| Zone **A**, | $\bar{x}$ +2σ to $\bar{x}$+3σ | 2 out of 3 points in zone A or above |
| Zone **B**, | $\bar{x}$ +1σ to $\bar{x}$+2σ | 4 out of 5 points in zone B or above |
| Zone **C**, | $\bar{x}$ + 1σ) | 8 in a row in zone C or above |
| Zone **C**, | $\bar{x}$ - 1σ) | 8 in a row in zone C or below |
| Zone **B** | $\bar{x}$ - 1σ to $\bar{x}$-2σ | 4 out of 5 points in zone B or below |
| Zone **A**, | $\bar{x}$ - 2σ to $\bar{x}$-3σ | 2 out of 3 points in zone A or below |
| | | Single point below lower control limit |

5.2 The standard error or standard deviation of a distribution of averages is $\sigma_{\bar{x}} = \dfrac{A_2 \bar{R}}{3}$.

5.3 An estimate of the standard deviation for the individual data values is $s = \sigma_{\bar{x}}\sqrt{n} = \dfrac{\bar{R}}{d_2}$.

This is a very useful statistic when comparisons must be made to the specification limits and the original individual measurements are not available.

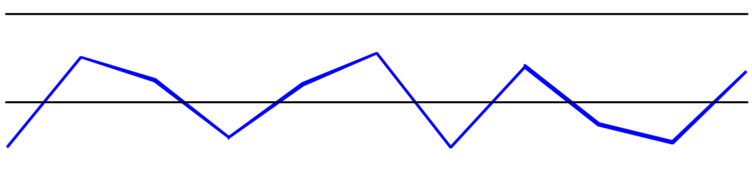
## 6.0 CONTROL CHART PATTERNS

When a control chart pattern is natural (follows normal curve pattern), there are no abnormal extraneous conditions in the process. When the pattern is not natural, something is present in the process that has an effect on the control chart pattern. All unnatural patterns need to be investigated to determine the cause. The chart itself will not determine the cause, it will just indicate whether the pattern is natural or unnatural. A chart that is in control will show natural patterns of variation.

Unnatural patterns tend to fluctuate too wide or they fail to balance around the centerline. The portrayal of natural and unnatural patterns is what makes the control chart a very useful tool for statistical process and quality control. When a chart is interpreted, we look for special patterns such as cycles, trends, freaks, mixtures, groupings or bunching of measurements, and sudden shifts in levels.

### 6.1 Natural Pattern

The points follow a standard normal curve pattern.



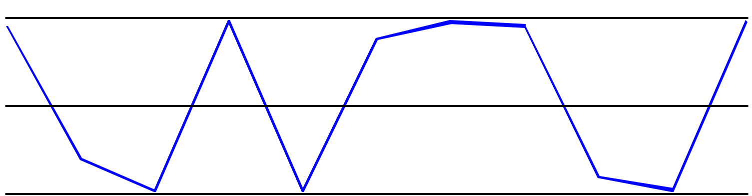
The characteristics of natural patterns:

- Most of the points are near the average
- A few points will approach the control limits
- Very rarely will points exceed the control limits

### 6.2 Mixtures

Most points fall near the control limits and few fall near the centerline.

.

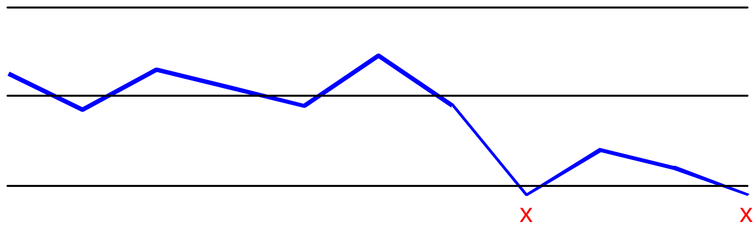


Possible causes of mixtures:

- Lots from different sources
- Differences in people performing checks
- Differences in test equipment
- Two or more different materials, machines, operators or gauges

## 6.3 Sudden Shift in Level

The points show a marked shift in one direction.

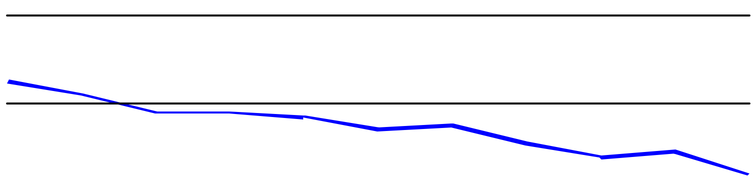



Possible causes of sudden shifts:

- New production equipment
- New people performing check
- New test sets
- New machine settings
- New supplier
- Inadequate or revised maintenance
- Change in equipment calibration
- Improved process capability

## 6.4 Trends

The points show a gradual shift in one direction.

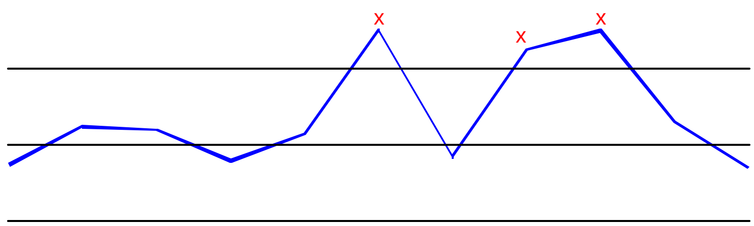


Possible causes of trends:

- Equipment wear
- Inadequate or improved maintenance
- Gradual introduction of new material
- Process improvement or deterioration

## 6.5 Grouping or Bunching

All or most points beyond the control limits appear quite close together.

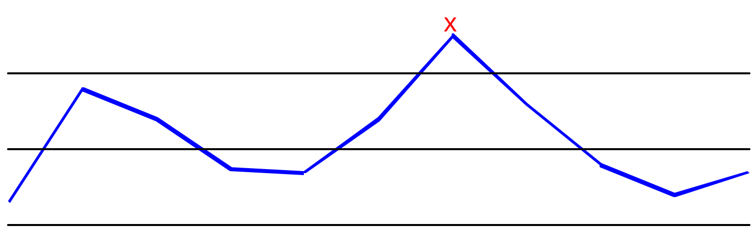



Possible causes of Grouping or Bunching:

- Different checkers or inspectors
- Plotting errors
- Change in test set calibration

## 6.6 Cycles

Cycles are short term repeating trends in the data.

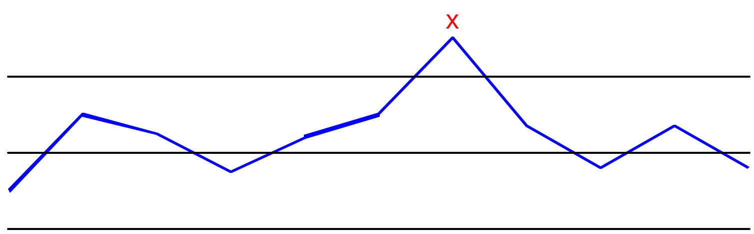



Possible causes of cycles:

- Different equipment
- Different checkers or inspectors
- Differences between suppliers
- Seasonal effects (temperature, etc.)

## 6.7 Freaks

Freaks are occasional occurrences that vary greatly from the normal pattern.

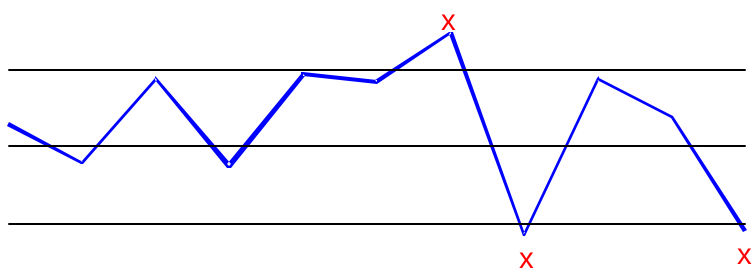



Possible causes of Freaks:

- Breakdown of equipment
- Accidental damage in handling
- Incomplete or omitted operation
- Wrong setting on machine
- Measurement error

## 6.8 Instability

Instability is characterized by unnaturally large fluctuations.  The pattern exhibits erratic ups and downs and is frequently out of control at both limits.




Possible causes of Instability:

- Mixture of material
- Machine in need of repair (unstable)
- Unstable test equipment
- Non-random sampling
- Untrained checker or inspector

## 7.0 FACTORS FOR COMPUTING LIMITS FOR X BAR AND R CHARTS

| Sample Size | For Averages | For Ranges | | Standard Deviation |
|---|---|---|---|---|
| **n** | **$A_2$** | **$D_3$** | **$D_4$** | **$d_2$** |
| 2 | 1.88 | 0 | 3.29 | 1.13 |
| 3 | 1.02 | 0 | 2.58 | 1.69 |
| 4 | .73 | 0 | 2.28 | 2.06 |
| 5 | .58 | 0 | 2.11 | 2.33 |
| 6 | .48 | 0 | 2.00 | 2.53 |
| 7 | .42 | 0 | 1.92 | 2.70 |
| 8 | .37 | .14 | 1.87 | 2.85 |
| 9 | .34 | .18 | 1.82 | 2.97 |
| 10 | .31 | .31 | 1.78 | 3.08 |

## 8.0 DATA SHEET FOR X BAR AND R CHARTS

| Sample Number L | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Measurement 1 | | | | | | | | | | |
| Measurement 2 | | | | | | | | | | |
| Measurement 3 | | | | | | | | | | |
| Measurement 4 | | | | | | | | | | |
| Measurement 5 | | | | | | | | | | |
| Total of Measurements | | | | | | | | | | |
| Average ($\bar{x}$) | | | | | | | | | | |
| Range (R) | | | | | | | | | | |

| Sample Number L | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Measurement 1 | | | | | | | | | | |
| Measurement 2 | | | | | | | | | | |
| Measurement 3 | | | | | | | | | | |
| Measurement 4 | | | | | | | | | | |
| Measurement 5 | | | | | | | | | | |
| Total of Measurements | | | | | | | | | | |
| Average ($\bar{x}$) | | | | | | | | | | |
| Range (R) | | | | | | | | | | |

Blank Page

## 9.0 CONTROL CHART TEMPLATE