CHAPTER 8

RELIABILITY

"It's a good thing to follow the first law of holes; if you are in one, stop digging."

Denis Healey

RELIABILITY

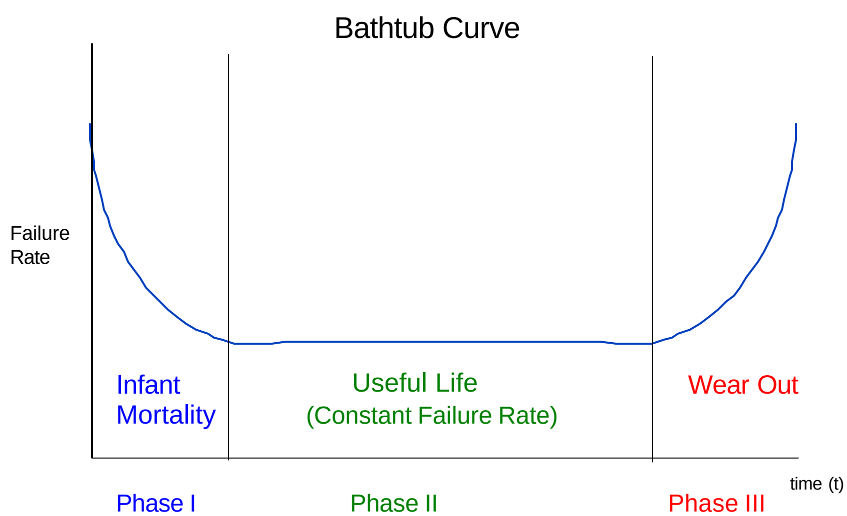
1.0 INTRODUCTION

The official definition of reliability is “the probability of a device performing its intended function under given operating conditions and environments for a specified length of time.” Using this definition, the probability of a device working for 100 hours and the reliability of a device designed to work for 100 hours are two ways to make the same statement.

The most basic method of achieving product reliability is through mature design. On new products, failure rates are determined under accelerated conditions and used to make reliability predictions. In complex assemblies, there may be hundreds of individual components that affect the reliability of the final product. Ideally, 100% reliability is desirable but that is not always possible to achieve. In products that affect human life, a high degree of reliability is absolutely necessary. These products have high quality components and are tested under extreme conditions. The reliability of a product, whether its an airplane or a computer, is dependent on the quality of its components. The subject of reliability introduces the factor of time in making probability calculations.

1.1 Life Curve for a Device

The three phases in the life of a product or device are described by a life cycle curve commonly referred to as the bathtub curve.



During the early life or infant stage of a device, failures occur more frequently than during the operating or useful life phase. During the latter part of the life of a device, the wear out phase, the frequency of failure is again high and rises rapidly. This can be verified by owners of twelve-year-old cars. During the useful life phase, the failure rates for most devices is constant. The length of the useful life is determined by the device or product. Light bulbs usually have a shorter useful life than car radios.

1.2 Reliability Terminology

Reliability calculations can only be made in the useful life phase (phase II) of a product or device. In the infant mortality and wear out phase there is too much variation in the failure rate to make reliability predictions. A product is usually in the customers or users possession after the initial problems (infant mortality) have been eliminated.

The main difference between the quality of a device and the reliability of a device is that reliability involves a time factor. Reliability is the probability of a device working for a specified interval of time.

In a quality problem, the question may be asked: What is the probability of one defective device or one failure in a sample of ten parts? The parts are either good or defective at the time that they are examined. In a reliability problem, the question may be: What is the probability that the device will work for 100 hours without a failure?

Failure rates and the subsequent reliability of devices are usually determined by a procedure called life testing. Life testing is the process of placing a device or unit of product under a specified set of test conditions and measuring the time it takes until failure. Life testing sampling plans are used to specify the number of units that are to be tested and for determining acceptability. The procedures for developing and using a life test sampling plan are almost the same as those used for acceptance sampling. The producer's and consumer's risks are specified, and an OC curve may be developed. The exponential distribution is used to find the probability of acceptance.

1.3 Failure Rate

The constant failure rate during the useful life (phase II) of a device is represented by the symbol lambda (λ). The failure rate is defined as the number of failures per unit time or the proportion of the sampled units that fail before some specified time.

$$\text{Failure rate} = \text{Lambda} = \lambda = f/n$$

Where f = the total failures during a given time interval and n = the number of units or items placed on test.

If 500 parts were placed on test and 21 failures were recorded between the sixth and seventh hour, then the failure rate $\lambda = 21/500 = .042$ failures per hour.

1.4 Mean Time between Failures

The mean time between failures or MTBF is the average length of life of the devices being tested. It is the reciprocal of the failure rate.

$$\text{Mean time between failure (MTBF)} = \text{Theta} = \theta = 1/\lambda$$

The mean time between failure for the above example = $1/\lambda = 1/.042 = 23.8$ hours.

1.5 Reliability Formula

The exponential distribution formula is used to compute the reliability of a device or a system of devices in the useful life phase. The exponential formula has its roots in the Poisson formula. Instead of np, the product λt is used. The exponential is the Poisson formula with x = 0. Reliability means the probability of zero failures in the specified time interval.

$$P(x) = \frac{e^{-np}(np)^x}{x!} = \frac{e^{-\lambda t}(\lambda t)^x}{x!} \text{ , for } x = 0,\ P(0) = e^{-\lambda t} = \text{Reliability}$$

$$\text{Reliability of a single device} = R = e^{-\lambda t}$$

Where t is the mission time and e is a constant value of 2.71828. The letter e represents the base of the natural system of logarithms. Most statistical calculators have an e^x key. Enter a one for x and the calculator will return the e value of 2.71828.

A sample of 450 devices were tested for 30 hours and 5 failures were recorded. The device is designed to operate for 1000 hours without failure. What is the reliability of the tested device?

$$\text{Failure rate} = \lambda = 5/(450)(30) = 5/13500 = .0003704$$

$$\text{Reliability} = e^{-\lambda t} = e^{-(.0003704)(1000)} = e^{-.3704} = .6905$$

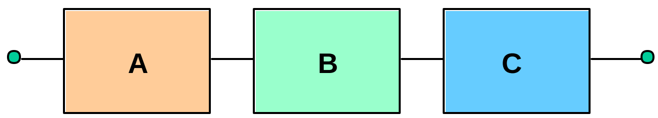
The probability of a device operating for 1000 hours without a failure is .69.05%.

2.0 RELIABILITY SYSTEMS

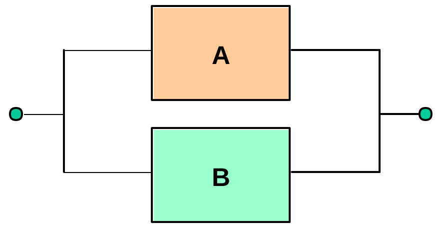
There are two basic types of reliability systems. They are series and parallel systems, similar to electrical circuits. In a series system, all devices must work for the system to work. In a simple parallel configuration, the system will work if at least one device works.

The reliability calculations for these systems are an extension of basic probability concepts. There are other configurations in addition to the two basic systems such as standby systems, switched systems, and combinations of each.

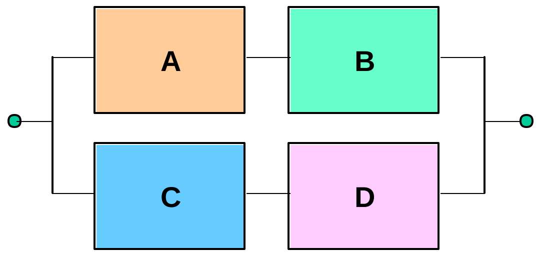
2.1 Series System



2.2 Parallel System



2.3 Combination System

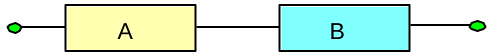


Example 1

What is the reliability of the series system shown below?

$\lambda_A = .001$, $\lambda_B = .002$, mission time (t) = 50 hours

For the system to work, both devices must work. If one device fails, the system fails.



R_A = reliability of device A = probability that device A will work for at least 50 hours

$R_A = e^{-\lambda_A t} = e^{-(.001)(50)} = .9512$

R_B = reliability of device B = probability that device B will work for at least 50 hours

$R_B = e^{-\lambda_B t} = e^{-(.002)(50)} = .9048$

R_S = reliability of system = probability that the system will work for at least 50 hours

$R_S = R_A \times R_B$

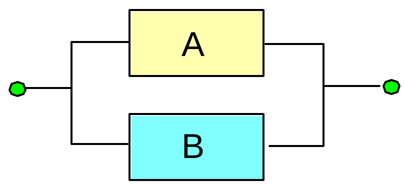
$R_S = .9512 \times .9048 = .8606$

Example 2

What is the reliability of the parallel system shown below?

$$\lambda_A = .001, \quad \lambda_B = .002 \quad t = 50 \text{ hrs}$$

For the system to work, one or both devices must work. The system will fail when both devices fail.



From example 1, $R_A = .9512$ and $R_B = .9048$

$R_{\bar{A}} = 1 - R_A$ = probability that device A fails and $R_{\bar{B}} = 1 - R_B$ = probability that device B fails.

R_S = reliability of system = $(R_A)(R_{\bar{B}}) + (R_{\bar{A}})(R_B) + (R_A)(R_B)$

$R_S = (.9512)(.0952) + (.04888)(.9048) + (.9512)(.9048)$

$R_S = .0906 + .0442 + .8606 = .9954$

Alternate solution: $R_S = 1 - (R_{\bar{A}})(R_{\bar{B}}) = 1 - (.0488)(.0952) = .9954$